Jet Propulsion Laboratory
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Reduced-order modeling and gravitational waveforms

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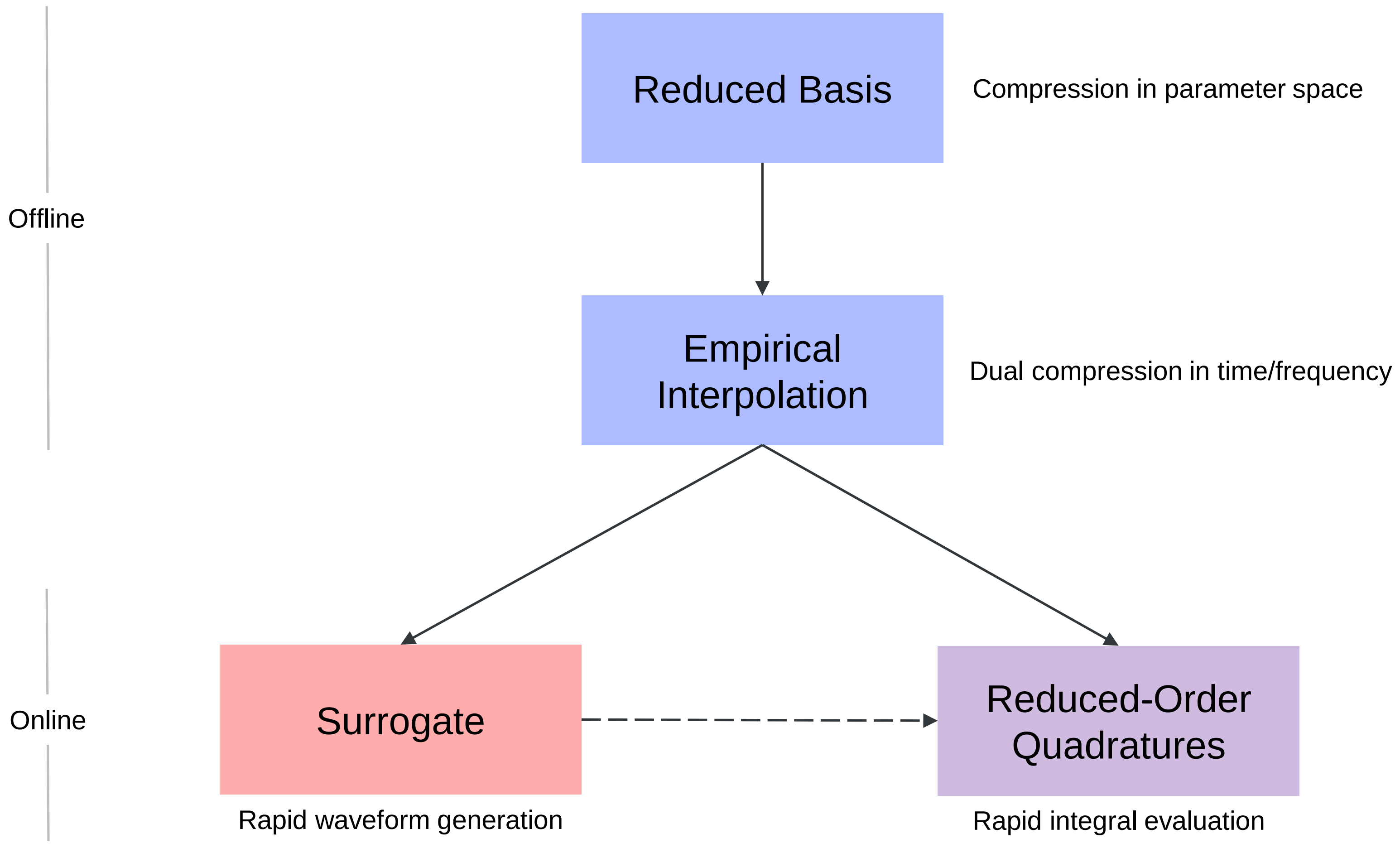
Motivation

- Generating waveforms can be expensive, time-consuming, and a bottleneck for practical data analysis applications
  - Template bank generation for gravitational wave searches (“curse of dimensionality”)
  - Multiple waveform queries for parameter estimation (e.g., with stochastic methods)
  - Parameter space mapping, exploration, and discovery (i.e., science!)
  - Accessibility to broader scientific communities and the public
- Goals: To cheaply and quickly predict gravitational waveforms that are otherwise prohibitively expensive to mass-produce.
  - Numerical relativity waveforms of compact binary coalescences
    - Can take weeks to months to complete one simulation and corresponding waveform
    - 3 points only in each of the 7 parameter dimensions requires 2187 simulations!
  - Extreme Mass Ratio Inspirals (EMRIs)
  - Continuous gravitational waves

Reduced-Order Modeling (ROM)

Overview



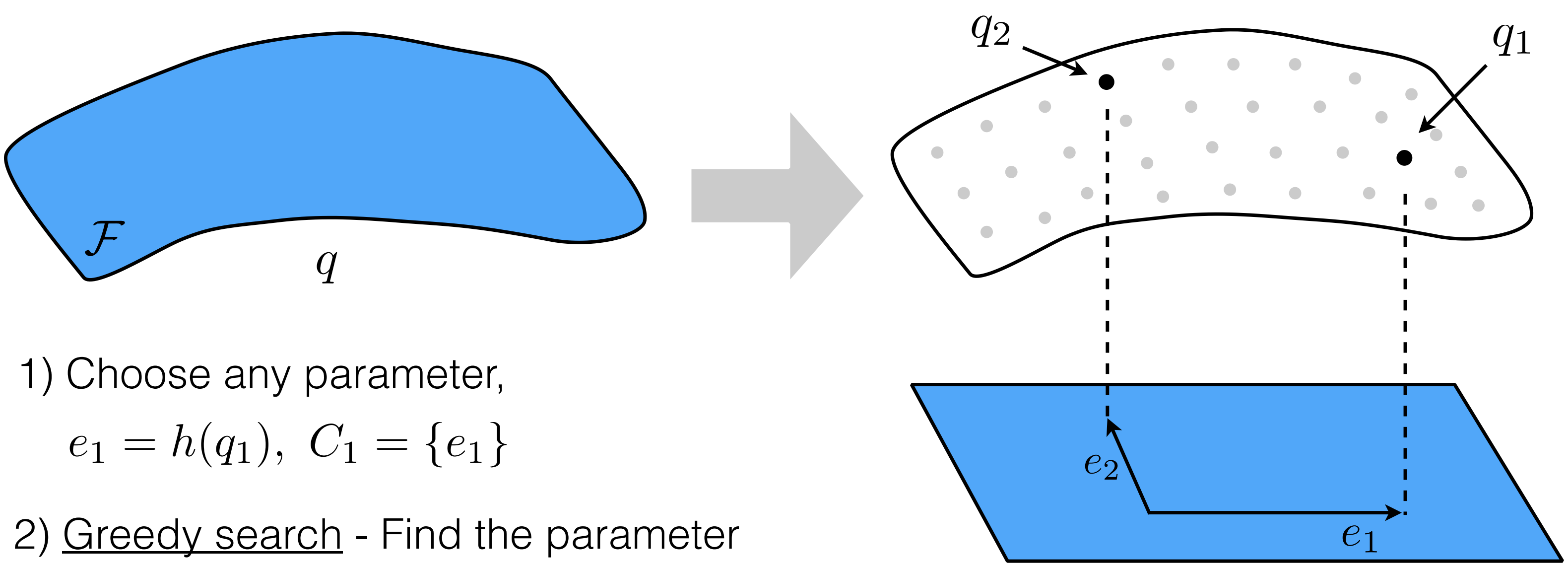
Reduced Basis

Basic Idea

Can find a linear approximation space that is nearly optimal

Set of waveforms $\mathcal{F}$

"Training space"



1) Choose any parameter,

$$e_1 = h(q_1),\ C_1 = \{e_1\}$$

2) Greedy search - Find the parameter that maximizes:

$$||h_q - P_1(h_q)||,\ P_1(h_q) = e_1\langle e_1, h_q\rangle$$

3) Orthogonalization to get basis vector e_2

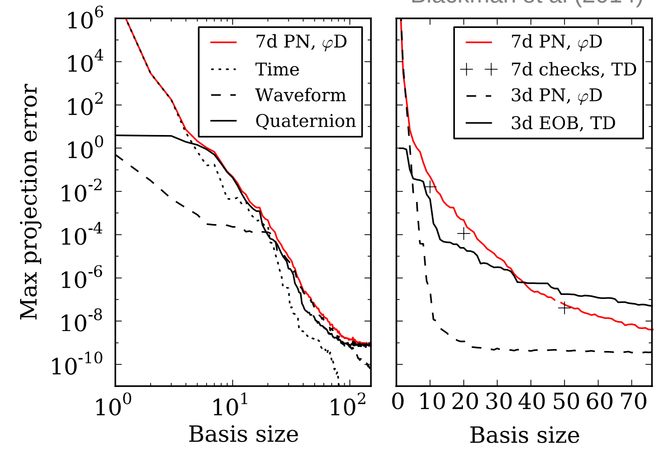
$$C_2 = \{e_1, e_2\},\ C_1 \subset C_2$$

Output:

1) "Most relevant" parameters and waveforms
2) A nested/hierarchical basis
3) Maximum projection errors converge exponentially

Reduced Basis

Lessons learned



- Arrange and transform the training data into a form that is smooth with parameter variations (i.e., "boring") [Blackman et al (2014)]
  - A 0PN waveform has a reduced basis with one element when parameterizing by phase instead of frequency

- The greedy algorithm is highly flexible and can adapt to many different types of strategies
  - Randomly resample the training set after each iteration [Blackman et al (2014)]
  - Use an error metric that is suitable to the problem and parameterization
  - Divide and conquer the training space (with random resampling) [Galley (unpublished)]

- Waveform representation by a reduced basis is robust to different detector PSDs [Field et al (2011)]

- Even if target ROM accuracy is small, it is typically useful to build the Reduced Basis with a much higher accuracy.

- Lower-accuracy waveform models (e.g., Phenom*, *EOB*) are helpful to inform for which parameters to run expensive simulations [Blackman et al (2015)]
  - Side-effect price to pay: More parameters tend to be selected than are actually needed in the end. (EOBNRHMv2 implied 22 simulations where the actual number is about 7.)

Empirical Interpolation

Barrault et al (2004)
Maday et al (2009)

Basic Idea

- A reduced basis is built to accurately span the waveform space
  - Therefore, all functions and functionals of waveforms can be accurately represented by the reduced basis
  - Including an interpolant built from the empirically found reduced basis

- Empirical interpolation is essentially the standard interpolation problem but constructed using the application-specific reduced basis instead of a generic basis (e.g., Chebyshev polynomials)
  - Interpolation nodes are selected by another greedy algorithm that minimizes the interpolation error

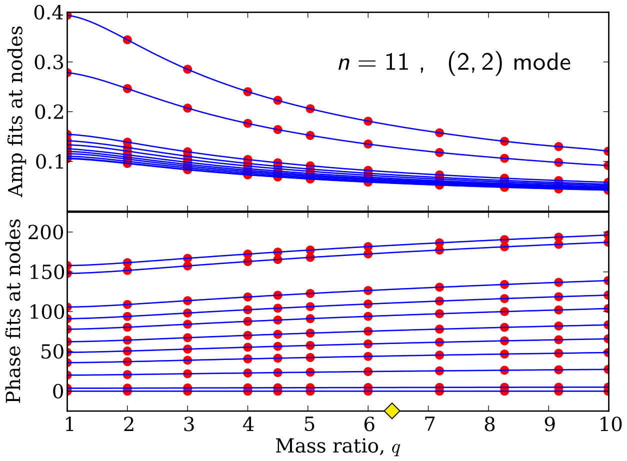
- Empirical interpolation errors are provably proportional to the maximum projection errors of the reduced basis.
  - In ROMs to date, empirical interpolation is within ~100x of the reduced basis representation error.

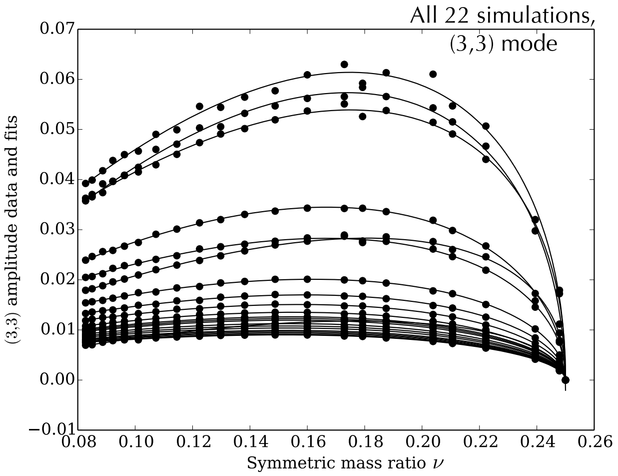
Surrogate

Basic Idea

- A “grid” of points in parameter space and physical (i.e., time/frequency) space is built for the features of the specific waveform family by RB and EIM

- Surrogate is constructed by fitting for the parameter variation at each empirical interpolation node (e.g., time)
  - (show equation), (highlight speed-up relative to nominal waveform generation times)





Reduced-Order Quadratures

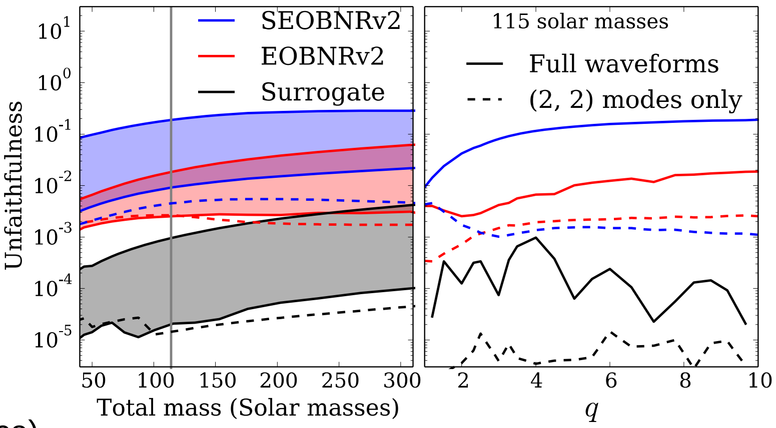
Basic Idea

- (Brief overview of what it is)
- (Parameter estimation applications, even for NR waveforms via its surrogate)
- (Mention small start-up cost before online stage)

Results

- Post-Newtonian inspirals
  - (non-spinning reduced basis)
  - (non-precessing reduced basis)
  - (quasi-normal mode ringdowns reduced basis)
- Effective One-Body surrogates
  - (EOBNRv2 and EOBNRHMv2)
  - (SEOBv?)
  - (TEOB)
- Numerical Relativity surrogates
  - (non-spinning BBH)
  - (non-precessing BBH)
  - (precessing BBH)

Blackman, et al (2015)



- EMRI surrogates
  - (black hole Green's function surrogate for scalar self-force)
- Continuous gravitational waves
  - (that recent paper by the Scottish folks – reduced basis for timing models)
- Reduced-Order Quadratures
  - (look-up Priscilla's papers to remind myself what they did exactly)

Short-comings and mitigations

- ROMs are accurate within the parameter domain of the training space
  - Caution must be taken when extrapolating outside the training space
  - Mitigations:
    - Increase the domain of the training space
    - Find a smoother waveform parameterization so that extrapolation is less severe

- Reduced-Order Quadratures rely on the linearity of the waveform itself
  - Smart (i.e., nonlinear) parameterizations for RB construction are not helpful here
  - Mitigations:
    - None
    - But, one can still use a surrogate (if needed) for rapid waveform generation in the integrand

- Offline generation of the training set can take a long time
  - Example: Took 2 years to generate 22 NR training waveforms for the non-spinning BBH surrogate
  - Mitigations:
    - Progress with time: Took ~2 years to generate (a few hundred) NR training waveforms for the precessing BBH surrogate
    - Adjust the training space sampling strategy for the Reduced Basis greedy algorithm

- ROM works best on a training set of $C^{infinity}$ functions
  - Can still work on C^n functions but yields a less compact reduced basis (e.g., EOBNRv2)
  - Mitigation:
    - (fill this out in a clear way)

Outlook

- (outlook)
- (rompy code public repo)

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